Department for Education

Mathematics

GCSE subject content and assessment objectives

Contents

Introduction

GCSE subject criteria set out the knowledge, understanding, skills and assessment objectives common to all GCSE specifications in a given subject. They provide the framework within which awarding organisations create the detail of their specifications, so ensuring progression from key stage 3 national curriculum requirements and the possibilities for development into advanced-level study.

Subject aims and learning outcomes

This document sets out the learning outcomes and content coverage required for GCSE specifications in mathematics. In subjects such as mathematics, where topics are taught in progressively greater depth over the course of key stage 3 and key stage 4, GCSE outcomes may reflect or build upon subject content which is typically taught at key stage 3. There is no expectation that teaching of such content should be repeated during the GCSE course where it has already been taught effectively at an earlier stage.

GCSE specifications in mathematics should provide a broad, coherent, satisfying and worthwhile course of study. They should encourage students to develop confidence in, and a positive attitude towards mathematics and to recognise the importance of mathematics in their own lives and to society. They should also provide a strong mathematical foundation for students who go on to study mathematics at a higher level post-16.

GCSE specifications in mathematics should enable students to:

1. develop fluent knowledge, skills and understanding of mathematical methods and concepts
2. acquire, select and apply mathematical techniques to solve problems
3. reason mathematically, make deductions and inferences and draw conclusions
4. comprehend, interpret and communicate mathematical information in a variety of forms appropriate to the information and context.

Students should be aware that mathematics can be used to develop models of real situations and that these models may be more or less effective depending on how the situation has been simplified and the assumptions that have been made. Students should also be able to recall, select and apply mathematical formulae (see Appendix).

Subject content

GCSE specifications in mathematics should reflect the aims and learning outcomes outlined above, and should include the knowledge, understanding and skills listed below, giving due consideration to the assessment objectives. The essential subject content outlined here provides the framework for developing a coherent study at GCSE.

This content sets out the full range of content for GCSE specifications in mathematics. Awarding organisations may, however, use any flexibility to increase depth, breadth or context within the specified topics or to consolidate teaching of the subject content.

Students can be said to have confidence and competence with mathematical content when they can apply it flexibly to solve problems.

The expectation is that:

- All students will develop confidence and competence with the content identified by standard type
- All students will be assessed on the content identified by the standard and the underlined type; more highly attaining students will develop confidence and competence with all of this content
- Only the more highly attaining students will be assessed on the content identified by **bold** type. The highest attaining students will develop confidence and competence with the **bold** content.

The distinction between standard, underlined and **bold** type applies to the content statements only, not to the assessment objectives or to the mathematical formulae in the appendix.

Scope of study

GCSE specifications in mathematics should require students to:

Number

Structure and calculation

1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
4. use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
5. apply systematic listing strategies **including use of the product rule for counting**
6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5***; estimate powers and roots of any given positive number***
7. calculate with roots, and with integer **and fractional** indices
8. calculate exactly with fractions, **surds** and multiples of π**; simplify surd expressions involving squares (e.g.** $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$**) and rationalise denominators**
9. calculate with and interpret standard form A x 10^n, where $1 \leq A < 10$ and n is an integer.

Fractions, decimals and percentages

10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 or $\frac{3}{8}$)**; change recurring decimals into their corresponding fractions and vice versa**
11. identify and work with fractions in ratio problems
12. interpret fractions and percentages as operators.

Measures and accuracy

13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
16. apply and interpret limits of accuracy**, including upper and lower bounds**

Algebra

Notation, vocabulary and manipulation

1. use and interpret algebraic notation, including:
    - ab in place of $a \times b$
    - $3y$ in place of $y + y + y$ and $3 \times y$
    - a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
    - $\frac{a}{b}$ in place of $a \div b$
    - coefficients written as fractions rather than as decimals
    - brackets
2. substitute numerical values into formulae and expressions, including scientific formulae
3. understand and use the concepts and vocabulary of expressions, equations, formulae, identities inequalities, terms and factors
4. simplify and manipulate algebraic expressions (including those involving surds **and algebraic fractions**) by:
    - collecting like terms
    - multiplying a single term over a bracket
    - taking out common factors
    - expanding products of two **or more** binomials
    - factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares**; factorising quadratic expressions of the form** $\boldsymbol{ax^2 + bx + c}$
    - simplifying expressions involving sums, products and powers, including the laws of indices
5. understand and use standard mathematical formulae; rearrange formulae to change the subject
6. know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**
7. where appropriate, interpret simple expressions as functions with inputs and outputs**; interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'.**

Graphs

8. work with coordinates in all four quadrants

9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular lines**; find the equation of the line through two given points, or through one point with a given gradient
10. identify and interpret gradients and intercepts of linear functions graphically and algebraically
11. identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically **and turning points by completing the square**
12. recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **exponential functions $y = k^x$ for positive values of k, and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size**
13. **sketch translations and reflections of a given function**
14. plot and interpret graphs (including reciprocal graphs **and exponential graphs**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
15. **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts**
16. **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point.**

Solving equations and inequalities

17. solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
18. solve quadratic equations (**including those that require rearrangement**) algebraically by factorising**, by completing the square and by using the quadratic formula**; find approximate solutions using a graph
19. solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph
20. **find approximate solutions to equations numerically using iteration**
21. translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.
22. solve linear inequalities in one **or two** variable**(s), and quadratic inequalities in one variable**; represent the solution set on a number line**, using set notation and on a graph**

Sequences

23. generate terms of a sequence from either a term-to-term or a position-to-term rule
24. recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 **or a surd**) **and other sequences**
25. deduce expressions to calculate the n^{th} term of linear **and quadratic** sequences.

Ratio, proportion and rates of change

1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
2. use scale factors, scale diagrams and maps
3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
4. use ratio notation, including reduction to simplest form
5. divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
6. express a multiplicative relationship between two quantities as a ratio or a fraction
7. understand and use proportion as equality of ratios
8. relate ratios to fractions and to linear functions
9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
11. use compound units such as speed, rates of pay, unit pricing, density and pressure
12. compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

13. understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; **construct and** interpret equations that describe direct and inverse proportion
14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
15. **interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts**
16. set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes.**

Geometry and measures

Properties and constructions

1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
2. use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
3. apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
5. use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
6. apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
7. identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)

8. **describe the changes and invariance achieved by combinations of rotations, reflections and translations**
9. identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
10. **apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**
11. solve geometrical problems on coordinate axes
12. identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
13. construct and interpret plans and elevations of 3D shapes.

Mensuration and calculation

14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
16. know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
17. know the formulae: circumference of a circle $= 2\pi r = \pi d$, area of a circle $= \pi r^2$; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids
18. calculate arc lengths, angles and areas of sectors of circles
19. apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures
20. know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacent}{hypotenuse}$ and $\tan\theta = \frac{opposite}{adjacent}$; apply them to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two **and three** dimensional figures
21. know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0^0, 30^0, 45^0, 60^0$ and 90^0; know the exact value of $\tan\theta$ for $\theta = 0^0, 30^0, 45^0$ and 60^0
22. **know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find unknown lengths and angles**
23. **know and apply $\text{Area} = \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle.**

Vectors

24. describe translations as 2D vectors
25. apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **use vectors to construct geometric arguments and proofs**

Probability

1. record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale
4. apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
5. understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
6. enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
7. construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
8. calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
9. **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.**

Statistics

1. infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use
3. **construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**

4. interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
    - appropriate graphical representation involving discrete, continuous and grouped data, **including box plots**
    - appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers**, quartiles and inter-quartile range**)
5. apply statistics to describe a population
6. use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Assessment objectives

Assessment Objectives		Weighting	
		Higher	Foundation
AO1	**Use and apply standard techniques** Students should be able to: • accurately recall facts, terminology and definitions • use and interpret notation correctly • accurately carry out routine procedures or set tasks requiring multi-step solutions	40%	50%
AO2	**Reason, interpret and communicate mathematically** Students should be able to: • make deductions, inferences and draw conclusions from mathematical information • construct chains of reasoning to achieve a given result • interpret and communicate information accurately • present arguments and proofs • assess the validity of an argument and critically evaluate a given way of presenting information Where problems require candidates to 'use and apply standard techniques' or to independently 'solve problems' a proportion of those marks should be attributed to the corresponding Assessment Objective	30%	25%
AO3	**Solve problems within mathematics and in other contexts** Students should be able to: • translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes • make and use connections between different parts of mathematics • interpret results in the context of the given problem • evaluate methods used and results obtained • evaluate solutions to identify how they may have been affected by assumptions made Where problems require candidates to 'use and apply standard techniques' or to 'reason, interpret and communicate mathematically' a proportion of those marks should be attributed to the corresponding Assessment Objective.	30%	25%

Appendix: Mathematical formulae

1. Formulae included in the subject content. Candidates are expected to know these formulae; they must not be given in the assessment.

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Circumference and area of a circle

Where r is the radius and d is the diameter:

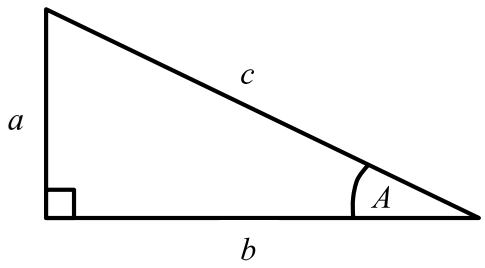
$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

Pythagoras's theorem

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$



Trigonometry formulae

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

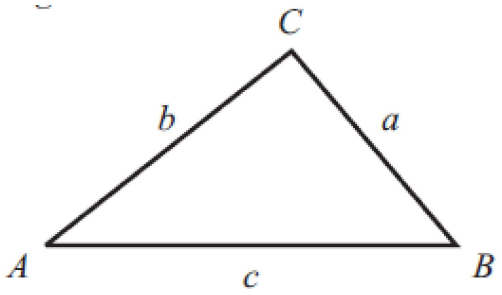
$$\sin A = \frac{a}{c}, \quad \cos A = \frac{b}{c}, \quad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\text{Area} = \frac{1}{2}ab\sin C$$



2. <u>The following formulae are not specified in the content but should be derived or informally understood by candidates. These formulae must not be given in the examination.</u>

Perimeter, area, surface area and volume formulae

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a+b)h$$

$$\text{Volume of a prism} = \text{area of cross section} \times \text{length}$$

Compound interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where $P(A)$ is the probability of outcome A and $P(B)$ is the probability of outcome B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)P(B)$$

3. Formulae that candidates should be able to use, but need not memorise. These can be given in the exam, either in the relevant question, or in a list from which candidates select and apply as appropriate.

Perimeter, area, surface area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

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Any enquiries regarding this publication should be sent to us at: www.education.gov.uk/contactus.

This document is also available from our website at: www.education.gov.uk/schools/teachingandlearning/qualifications/gcses.

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